**Slide 1: Intro**

**Slide 2: Some background**

On laser cooling… what happens if particles do not have closed cycling transitions and have narrow linewidth? Why is it challenging to cool particles with narrow linewidth?

**Slide 3: Basic mechanism**

The basic mechanism for laser cooling by sawtooth-wave adiabatic passage (or SWAP!) is rather intuitive because is it based on the Doppler effect and adiabatic transfer of atoms to and from a long-lived optically excited state.

Consider the model 2-level system as shown. The internal degrees of freedom are the two basis states (e, for excited, and g, for ground). The external degree of freedom is the motion of the atom – it has momentum **p**. The atom is initially in the ground state and is traveling with velocity **v** in the lab frame in the presence of two counter-propagating laser beams whose frequency is swept across the atomic transition frequency of the atom. The sweep is repeated so that there is a sawtooth pattern in time. The light as wavenumber **k** and frequency **ωL**. The full range of the sweep is **Δs**.

The cooling mechanism can be explained as follows. In the reference frame of the atoms, both beams start below the resonance with the atomic transition. However, Doppler shifts due to the relative motion of the atom with the beam cause the beam propagating against the atom velocity to be upshifted by **kv** while the co-propagating beam downshifted by **kv**. As a result, when the frequency of the light (in the lab frame) is swept up, the counter-propagating beam becomes resonant first and adiabatically transfer the atom from the ground state to the excited state.

Since lifetime of the excited state is large (remember, we’re interested in a narrow linewidth transition here), the atom remains in the excited state until the co-propagating beam sweeps over the resonance and adiabatically transfers the atom back to the ground state.

Through this process, the atom absorbs one photon from the counter-propagating beam, changing its momentum by **-hbar k**, and emits (via stimulated emission) a photon into the co-propagating beam, changing its momentum by another **-hbar k**. In total, the atom loses **2 hbar k** against its motion. Since the frequency jumps and the end of the sweep is diabatic, the process repeats! Applying this argument for an atom initially moving in the other direction, we see that the atom is decelerated either way!

**Slide 4: Requirements**

As mentioned, the full range of the sweep is **Δs**. This is sufficiently small such that the wavenumber **k** may be treated as constant (and therefore the recoil frequency **ωr** is also constant). Each laser interacts with the atom with Rabi frequency **Ω** which is larger than the spontaneous emission rate **γ** for the two-level system.

From the description of the cooling mechanism, we must make sure the parameters involving the light and the sweep are appropriate. Specifically, meeting the time-ordering and adiabatic condition is critical. In addition, we must make sure that the atoms are in the ground state at the beginning of each sweep, since atoms in the excited state will *gain* momentum from this scheme.

The relevant parameters for this scheme are as follows: the sweep range **Δs**, the sweep duration **Ts**, and the Rabi frequency associated with the atom-light interaction.

If the particle is in the excited state at the beginning of a sweep, it will *gain* rather than lose momentum. As a result, we must rely on spontaneous emission to ensure that the particle begins a sweep in the ground state. This is enforced by making the sweeping range sufficiently large so that the particle spends more time *outside* of the two resonances. This condition also enforces the condition that the sweeping range is sufficiently large so that both beams can achieve resonance with the atom.

Separately, we do not want the particle to have a large probability of decay when it is between the resonances. Therefore, we require that the **τe** be much less that the lifetime of the excited state.

Finally, we want the Rabi frequency and the sweep rate **α** to satisfy the well-known adiabatic condition. Here we have defined the adiabaticity parameter **κ** to be the ratio **Ω^2/α**. We want this ratio to be at least unity to obtain a substantial probability for the atom to be the excited state after interacting with the first beam.

Now, as an aside, I just wanted to emphasize that this cooling scheme not only reduces the velocity of the atoms but also increases the phase-space density, by means of a small amount of spontaneous emission. In the absence of spontaneous emission, the atom would find itself on a heating trajectory after initial cooling. The presence of spontaneous emission ensures that the atom is preferentially in the ground state at the beginning of each sweep. This breaks time reversal symmetry, enabling cooling and phase space compression to occur.

**Slide 5: Validity**

Not surprisingly, this cooling scheme is not always valid. Atoms in different velocity classes behave differently when interacting with the SWAP light.

It turns out that the atoms must be travelling sufficiently fast to interact with the counter-propagating beams in the manner described before. This is called the *high-velocity regime*. The reason is as follows: As the atom is slowed to near-zero velocity, the Doppler shift becomes small compared to the Rabi frequency **kv ~** **Ω**, and the condition for deterministic time-ordering of adiabatic transfers from the two beams (as described in the ideal scenario above) fails to hold. One can see the narrowing of the resonance window in this graph on the left from a simulation. When this happens, simple picture where the atom is sequentially excited and deexcited is no longer valid, and the probability that the atom remains in the excited state after a sweep becomes appreciable (as opposed to vanishing). And as we have seen, an atom in initially in the excited state will *gain* momentum following a sweep – this is undesirable.

So, we must be in a *high-velocity regime*. To define this, we must look in detail the time-ordering of the excitation-deexcitation process.

As shown, there are two time-related quantities here: **Tjump**, the time it takes to adiabatically transfer a particle between its internal states, and **Tres**, the time interval separating the two resonances. The total excited time, **Te**, is the sum of **Tjump** and **Tres.**

Since roughly half of each **Tjump** overlaps with **Tres**, the *high-velocity regime* is the range of velocities for which **Tjump < Tres.** Since we can relate these quantities to the sweep parameters, we obtain the following relation: **|Ω0| < |kv - 2ωr|.**

You might be worried that once atoms are cooled below the high-velocity regime, they get heated up – However this is only true if the evolution of the system is entirely unitary. It turns out that spontaneous emission helps here. In the presence of spontaneous emissions, the atom preferentially starts in the ground state at the beginning of each sweep. Like I said before, this breaks tie-reversal symmetry and phase space compression can occur.

**Slide 6: Force and capture range – without dissipation**

We first look at the case where we do not include effects of spontaneous emission. To describe characterize force without dissipation (aka conservative force), the authors consider the change in the atom’s rms momentum due to a single sweep. The reason they do as opposed to looking at the expectation value of momentum is to exclude effects of *Bragg oscillations.* (explain this!)

There are three relevant velocity regimes.

* If **|kv|> Δs/2** then the lasers are never on resonance with the atoms. This is not relevant.
* If **|kv|< |Ω0|** then as we can see the dynamics is more complicated as the momentum approaches zero. We can also explicitly see that a particle initially in the ground state with zero momentum will gain momentum from a sweep. This is because it will first get transferred to the excited state, which according to what was described before, gains momentum. So, in order to minimize this effect, we require the first condition on slide 4, which on average resets the particle to the ground state for the next sweep via spontaneous emission.
* If **|Ω0|<|kv|< Δs/2** then we’re (consistently) in the high-velocity regime as discussed before. The dynamics here is as expected. Here, we pay attention to the difference between diabatic and adiabatic cases. In the diabatic sweep, the particle receives a small impulse but fails to return to its initial internal state. In contrast, when the sweep is adiabatic, there is an impulse of size **|p\_rms| = 2 hbar k**, as expected and significant return to the initial internal state.

**Slide 7: Force and capture range (cont.) – with dissipation**